Rent appropriation and sustained growth

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Abstract

This paper demonstrates that the introduction of imperfect competition into the labor market can solve the problem isolated by Jones and Manuelli (Journal of Economic Theory, 1992, 58, 171–197), and Boldrin (Journal of Economic Theory, 1992, 58, 198–218), that in economies with convex technologies and finitely lived agents, real wages may not grow fast enough for unbounded growth to be sustained. I show that if wages are determined by a bargaining solution, and if the bargaining power of the workforce is sufficiently high (if they appropriate a sufficiently large proportion of rents), then growth is unbounded. Moreover, the growth path generated by such an economy may improve the welfare of all generations apart from the initial old.

Keywords: Sustained growth; Factor distribution

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1. Introduction

This paper studies how the distribution of rents from production determines the ability of the economy to grow. Jones and Manuelli (1992) and Boldrin (1992) have shown that in economies where individuals are finitely lived, sustained growth is not possible with convex technologies in a competitive equilibrium. The intuitive reason for this is that the marginal product of labor and, therefore, wages do not rise sufficiently fast to purchase the expanding capital stock. At some point this phenomenon brings growth to a halt.

I show how the introduction of imperfect competition into the labor market can resolve this difficulty. I show that if the wage rate is determined by a bargain then, if the marginal productivity of capital is bounded from below, and the bargaining power of workers is sufficiently large, the economy will experience unbounded growth. The result can be re-stated...
in terms of a condition on the marginal productivity of capital, given a level of ‘bargaining power’. The imperfectly competitive wage determination process fundamentally changes the model in a way that makes it straightforward to state such a sufficient condition for growth to be unbounded.

Jones and Manuelli (1992) propose various alterations to the basic model to allow growth to be sustained. These include various bequest schemes, a two-sector model where the relative price of capital goods can fall, government transfer schemes from old to young agents, and adding external effects to technology. Caballé and Manresa (1993) and Engel and Kletzer (1992) have developed these themes. The first paper considers a model with private decreasing returns but social constant returns due to a positive external effect. The external effect generates rents and the authors demonstrate that the distribution of these is crucial in determining whether or not the economy grows. Engel and Kletzer focus on how the distributional impact of fiscal policies affects the growth rate.

2. The economy

2.1. The competitive economy

Consider an overlapping generations model without bequests, as in Diamond (1965). Time is discrete. There are two goods in the economy. A produced good (with price unity), which can be consumed or used as capital, and labor. Each individual lives for two periods. In each period a generation is born consisting of a continuum of identical agents distributed uniformly on the unit interval with a constant population mass of one. A representative agent is denoted $i \in [0, 1]$, with $\int_0^1 di = 1$.

Each individual of each generation is endowed with one unit of labor when young and none when old. All old age consumption must be sustained from savings. Each has identical preferences defined over consumption in both periods of life. Preferences are represented by a homothetic utility function, $u(c_t(i), c_{t+1}(i))$, where $c_t(i)$ denotes consumption of agent $i$ born in period $t$, and $c_{t+1}(i)$ is consumption of such an agent in $t + 1$.

Assumption 1. $u : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}, u \in C^2$, and concave with the following properties: $u_1 > 0$, $u_{11} < 0$, $u_2 > 0$, $u_{22} < 0$. I assume that young and old consumption are both normal goods and gross substitutes. Furthermore, I assume the Inada conditions at the origin, $\lim_{c_t \to 0} u_1 = \infty$, $\lim_{c_{t+1} \to 0} u_2 = \infty$.

A young agent supplies his or her labor endowment to the labor market at the competitively determined wage rate $w_t$, and chooses savings, $s_t(i)$. Savings bear a gross rate of return of $1 + r_{t+1}$. Each young agent solves the problem:

$$\max_{c'_t(i), c'_{t+1}(i)} u(c'_t(i), c'_{t+1}(i))$$

subject to $c'_t(i) + s_t(i) = w_t$ and $c'_{t+1}(i) = (1 + r_{t+1})s_t(i)$. 
The solution is a savings function, \( s_i(i) = s(w_t, r_{t+1}) \). Homotheticity of preferences implies that this can be written as \( s_i(i) = s(r_{t+1})w_t \), where \( s: \mathbb{R}_+ \rightarrow (0, 1) \) is monotonically increasing (under the gross substitutes assumption). The Inada conditions guarantee that the range is the interior of the unit interval. Since all agents are identical, \( s_i(i) = s_i(i') = s_t \) for all \( i, i' \in [0, 1] \). Aggregate saving is \( \int_0^1 s_t(r_{t+1})w_t \, dt = s(r_{t+1})w_t \). The indirect utility function can be written as a linear function of income, denoted \( v(r_{t+1})w_t \).

There is a continuum of identical firms of unit mass distributed on the unit interval and indexed by \( j \in [0, 1] \). Each firm possesses a linearly homogeneous production function for converting labor and capital into output. Let \( y_j(j) = F(k_j(j), n_j(j)) \) denote output of firm \( j \), where \( n_j(j) \) is employment and \( k_j(j) \) is capital hired by firm \( j \).

**Assumption 2.** \( F: \mathbb{R}_+ \times [0, 1] \rightarrow \mathbb{R}_+ \), \( F \in C^2 \), and concave with the properties: \( F_k > 0 \), \( F_{kk} < 0 \), \( F_n > 0 \), \( F_{nn} < 0 \), and \( F(k, 0) > 0 \), \( F(0, n) = 0 \). I also assume the Inada condition at the origin, \( \lim_{k \rightarrow 0} F_k \rightarrow \infty \).

**Assumption 3.** The marginal product of capital is bounded below by some \( A > 1 \).

The property \( F(k, 0) > 0 \) is an inevitable feature of linearly homogeneous technologies that have a lower bound on the marginal product of capital. \( k_i(j) = k_i(j') = k_i \) for all \( j, j' \in [0, 1] \), and similarly for production and employment. Then since \( k_i = \int_0^1 k_i \, dj \), I use \( k_i \) to refer both to an individual firm and to the aggregate. Assumption 3 is the critical one in producing long-run growth. I use the following production function, introduced by Kurz (1968) and used by Jones and Manuelli (1990), to illustrate the results of the paper, \( y_i = Ak_i + k_i n_i \).

Each firm faces competitive factor and output markets so that \( w_i = F_n(k_i, n_i) \) and \( r_i = F_k(k_i, n_i) \). Equilibrium in the labor market implies \( \int_0^1 n_i \, dj = 1 \). I assume that the capital stock in any period is just the savings from the previous period, \( k_{t+1} = s(r_{t+1})w_t \).

**Proposition 1.** (Jones–Manuelli, Boldrin). There are no competitive equilibria of the economy with convex technology that exhibit unbounded growth.

With the specific production function, \( w_i = (1 - \alpha)k_i^\alpha n_i^{-\alpha} \). Using labor market clearing one finds \( w_i/k_i = (1 - \alpha)k_i^{\alpha-1} \), which clearly goes to zero as \( k_i \) goes to infinity.

### 2.2. The imperfectly competitive economy

I suppress the competitive capital market and assume that individuals become equity holders in old age. A young worker uses his or her capital to provide equity to a firm run by entrepreneurs whose objective is to maximize profits. The entrepreneurs hire labor and since the equity holders own all the assets of the firm they claim residual profits as the return on capital (distributed as dividends).

Firms and workers bargain over both wages and employment. I assume an efficient Nash bargain as in McDonald and Solow (1981). Since workers do not suffer disutility of labor,
efficient bargaining over employment intuitively implies \( n_t = 1 \). I assume that employment, denoted \( n_t \), is shared equally so that each worker would supply a proportion of his or her time if the bargain implied \( n_t < 1 \). I then prove that they, in fact, wish to set employment at unity.

I assume that if there is disagreement in bargaining then both firm and workers get an outside option normalized to zero. The model is easily extended to allow tax financed unemployment benefits (see Bean and Pissarides, 1993). The wage and employment level is determined in each period to solve the problem:

\[
\max_{w_t, n_t} \left\{ (w(r_{t+1})w_{n_t})^\beta (F(k_t, n_t) - w_{n_t})^{1-\beta} \right\},
\]

subject to \( n_t \leq 1 \). This problem is concave, so the optimizing choice is unique. \( \beta \in (0, 1) \) is a measure of bargaining power. If \( \beta = 1 \), then all the bargaining power accrues to the workforce.

**Proposition 2.** The Nash bargain implies \( n_t = 1 \), full employment.

**Proof.** The first-order conditions for the maximization of the Nash product with respect to \( w_t \) and \( n_t \) respectively are:

\[
w_t(1-\beta)n_t = \beta(F - w_{n_t}),
\]

\[
(1-\beta)(F_n - w_t) + \frac{\beta}{n_t} \geq 0 \text{ or } 0, \quad \text{if } n_t < 1.
\]

Substituting the first into second implies \( F_n = 0 \) if \( n_t < 1 \). Under Assumption 2, \( F_n > 0 \) so it must be true that \( n_t = 1 \) and \( F_n > 0 \). \( \square \)

Substituting \( n_t = 1 \) into the first first-order condition, it can be solved for the wage \( w_t = \beta F(k_t, 1) \). This implies that the return on capital is \( r_t = (1-\beta)F(k_t, 1)/k_t \).

**Proposition 3.** If the bargaining power of the workforce is sufficiently large then the economy can exhibit sustained growth.

**Proof.** Following the same logic as Proposition 1, it is necessary to establish that \( w_t/k_t > 1 \) for all \( t \). Now, \( w_t/k_t = \beta F(k_t, 1)/k_t \). As \( k_t \to \infty \) the assumption that \( F_k = A \) implies that \( \beta F(k_t, 1) \geq \beta A k_t \). Hence, sustained growth is possible if \( \beta A k_t > 1 \) or if \( \beta > 1/A \), for a particular fixed productivity parameter \( A \). \( \square \)

If \( \beta = 1 \) the economy cannot grow since the return to saving is zero. Imperfect competition in the labor market fundamentally changes the nature of the wage determination process. Inspecting \( \beta A > 1 \) shows that, with bargained wages, it is possible to state Proposition 3 in terms of the bound on the marginal product of capital for fixed \( \beta \in (0, 1) \). Proposition 4 below shows that as long as \( \beta \) is interior to the unit interval then growth will be unbounded as long as \( A \) is sufficiently high. While this seems the most convenient way to state the result, it could be stated in terms of a condition on \( \beta \) for fixed \( A > 1 \).
While Proposition 3 establishes that the economy may exhibit sustained growth, it does not establish that it does so. In order to verify this I need to examine saving behavior more carefully. The dynamics of the economy are determined by the following difference equation:

\[ k_{t+1} = s((1 - \beta)F(k_{t+1}, 1)/k_{t+1})\beta F(k_t, 1). \] (1)

This implicitly defines \( k_{t+1} = \Psi(k_t) \). \( \Psi \) is single-valued since saving is an increasing function of the interest rate. Now, \( k_{t+1}/k_t = s(r_{t+1})(\beta F(k_t, 1)/k_t) \). For sustained growth to occur the right-hand side must be greater than unity for all \( t \). Now \( \beta F(k_t, 1)/k_t \geq \beta A > 1 \) by hypothesis; however, \( s(r_{t+1}) \in (0, 1) \). Thus, given \( \beta A > 1 \), a sufficient condition for growth will be that the saving rate is sufficiently high. Denote: (i) \( r^c_t = F_k(k_t, 1) \) (the competitive interest rate), and (ii) \( r^b_t = (1 - \beta) F(k_t, 1)/k_t \) (the ‘bargained interest rate’). With the specific production function these become \( r^c_t = A + \alpha k_t \) and \( r^b_t = (1 - \beta)[A + k_t^{\alpha - 1}] \). If growth occurs for the asymptotic interest rate then it occurs a fortiori for transitional interest rates, hence growth is guaranteed if the following inequality holds:

\[ s((1 - \beta)A) > 1/A\beta. \] (2)

**Proposition 4.** Assume \( \beta \in (0, 1) \). Since the savings function is monotonically increasing in the interest rate there exists an \( A \), denoted \( A^* \), such that the economy grows unboundedly.

**Proof.** Under Assumption 1 the left-hand side of (2) is monotonically increasing in \( A \), while the right-hand side is decreasing in \( A \). Therefore, by continuity, there exists an \( A \) such that (2) holds as an equality. Let \( A^* = \tilde{A} + \epsilon \), where \( \epsilon > 0 \) is some small number. As \( A \) is increased the level of bargaining power necessary to sustain growth falls monotonically so the conditions of Proposition 3 can easily be satisfied. □

The economy has an asymptotic steady-state constant growth rate of \( s((1 - \beta)A)/A\beta \).

Notice, \( 0 = \Psi(0) \), so the origin is a fixed point (a stationary equilibrium). This map has the derivative:

\[ \frac{dk_{t+1}}{dk_t} = \frac{s(r_{t+1})(1 - \beta)\beta F(k_t, 1)}{1 - s'(r_{t+1})(1 - \beta)\beta F(k_t, 1)[F_k(k_{t+1}, 1) - F(k_{t+1}, 1)]/(k_{t+1})^2}. \] (3)

The numerator is positive for all \( k_t \), and since \( k_{t+1}F_k(k_{t+1}, 1) - F(k_{t+1}, 1) = -F_n(k_{t+1}, 1) < 0 \) the denominator is also positive. Hence \( \Psi' > 0 \). As \( k_t \to 0 \) then from (1), \( k_{t+1} \to 0 \). Now, \( \lim_{k_{t+1} \to 0} r_{t+1} = \lim_{k_{t+1} \to 0} (1 - \beta)F_k(k_{t+1}, 1) = \infty \), by Assumption 2 and L'Hôpital's rule. Therefore, as \( k_t \to 0 \) the numerator of (3) goes to infinity by Assumption 1 on saving behavior.

Consider the denominator. Notice that the term in large brackets can be re-written using Euler's theorem as \( -F_n/(k_{t+1})^2 \). As \( k_{t+1} \to 0 \) this ratio goes to minus infinity. The value of \( F(k_t, 1) \) is going to zero, however. In general, it is impossible to say what the limit of this product is. However, in the case of the specific technology I have used to illustrate the results, the product goes to minus infinity. Notice that \( s' > 0 \) and increasing as \( k_t \to 0 \). This suggests that the denominator goes to plus infinity also. To evaluate the derivative at the origin we need to know what goes to infinity faster. This turns out to be impossible to determine in general.
In a general competitive model the standard conditions on preferences and technology are not sufficient to guarantee \( \Psi'(0) > 1 \) (Galor and Ryder, 1989). The above discussion shows that the situation is pretty much the same in the present model.

Asymptotically, saving is not a function of the capital stock since the interest rate converges to a constant, hence \( \frac{dk_{t+1}}{dk_t} = s((1 - \beta)A)\beta A > 1 \). Intuitively, \( \Psi' \) is initially large and one might hope that it falls monotonically, asymptoting to the constant value. Unfortunately, it is also difficult to find sufficient conditions for behavior such as this (Galor and Ryder, 1989). That such behavior is not pathological is established by the following example (which does not strict satisfy Assumption 2).

Assume \( u(c'_t, c'_{t+1}) = \gamma \log c'_t + (1 - \gamma) \log c'_{t+1} \), and the production function is as in the example used so far. In this case, (1) becomes, \( k_{t+1} = (1 - \gamma) \beta(Ak_t + k_t^\alpha n_t^{\alpha - 1}) \). Hence, \( \frac{dk_{t+1}}{dk_t} = (1 - \gamma) \beta(A + \alpha k_t^{\alpha - 1}n_t^{\alpha - 1}) > 0 \). This example satisfies \( \Psi'(0) > 1 \) (in fact, \( \lim_{k_t \to 0} \Psi' = \infty \)). Also, \( \Psi'' = (1 - \gamma) \beta(\alpha - 1)k_t^{\alpha - 2}n_t^{\alpha - 1} < 0 \), which follows from \( \alpha - 1 < 0 \).

\( \Psi'(0) > 1 \) implies that the stationary equilibrium at the origin is not an attractor. As long as the economy starts with some positive amount of capital it will grow for ever.

2.3. Welfare

The result of this paper is not an example of the theory of second best. The competitive equilibrium of the economy of Subsection 2.1 is Pareto optimal even though it does not exhibit sustained growth. Similarly, the imperfectly competitive equilibrium of Subsection 2.2 is Pareto optimal. These two paths cannot be ranked. Endowing young agents with bargaining power hurts the initial old generation at time zero. An interesting question is whether this is the only generation that would be hurt by a movement from a competitive labor market to an imperfectly competitive one. Although all young agents appropriate a larger share of rents when young, the equilibrium interest rate will change. Asymptotically, \( r_i^c > r_i^b \). This will be true along the whole equilibrium path if \( \beta \) and \( \alpha \) are sufficiently large. Using the specific technology: \( r_i^c - r_i^b = A\beta + k_t^{\alpha - 1}(\alpha + \beta - 1) \) and \( w_i^b - w_i^c = A\beta + k_t^{\alpha - 1}(\alpha + \beta - 1) \). A sufficient condition for \( r_i^c - r_i^b > 0 \) and \( w_i^b - w_i^c > 0 \) is \( \alpha + \beta > 1 \). This seems to be the main case of interest and I concentrate on it in Proposition 5.

A sufficient condition for the welfare of a generation to improve is that the budget set of each cohort expands. I now establish that this happens if, for fixed \( A \) and \( \beta \) satisfying Proposition 3, the initial capital stock is large enough.

Proposition 5. If \( A\beta > 1 \), \( r_i^c - r_i^b > 0 \) and \( w_i^b - w_i^c > 0 \), and if the initial capital stock of the economy is sufficiently large, then the welfare of all generations apart from the initial old increases in moving from competitive wage determination to bargaining.

Proof. Consider the budget set of a representative agent born at \( t \). \( w_i^b > w_i^c \), so the intercept of the budget constraint with the horizontal axis moves to the right. To establish that welfare improves, it suffices to establish

\[ (1 + r_i^{b+1})w_i^b > (1 + r_i^{c+1})w_i^c \]  (*),
or \((1 + (1 - \beta)F(k_{t+1}, 1)/k_{t+1})\beta F(k_t, 1) > (1 + F_k(k_{t+1}, 1))F_n(k_t, 1)\). Now \(k_{t+1} = sw_t\). Since \(s \in (0, 1)\) it suffices to establish \(\beta F(k_t, 1) + (1 - \beta)F(k_{t+1}, 1) > (1 + F_k(k_{t+1}, 1))F_n(k_t, 1)\). Therefore, it suffices to establish \(F(k_t, 1) > (1 + F_k(k_{t+1}, 1))F_n(k_t, 1)\) since \(F(k_{t+1}, 1) > F(k_t, 1)\). Euler's theorem for homogeneous functions shows that this inequality implies \(k_t/F_n(k_t, 1) > F(k_{t+1}, 1)/F_k(k_{t+1}, 1)\). Since \(F_k(k_{t+1}, 1) \equiv F_k(k_{t-1}, 1)\), it suffices to show \(k_t/F_n(k_t, 1) \equiv 1\). Since the left-hand side of this last inequality goes to infinity as \(k_t \rightarrow \infty\) by continuity, there exists some \(k^*_t\) such that inequality (*) holds and only the initial old generation is made worse off by the introduction of bargaining.

To understand the condition, notice that if it is positive, \(w^b_t - w^c_t\) is increasing in \(k_t\). The ratio \(r^b_t/r^c_t\), while less than one, is increasing in \(k_t\). Geometrically, for low capital stocks, the budget constraint under bargaining intersects the competitive constraint. As the capital stock rises, the bargained constraint becomes steeper and its horizontal intercept increases relative to the competitive one (it is this phenomena that allows growth to be sustained). At some point the bargained constraint moves uniformly above the competitive budget constraint.

References


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